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ABSTRACT

The way that graphs can be used to make calculations that are commonly used by institutional researchers is described using specific examples, and the technique of constructing computational graphs (and nomographs) is outlined. It is shown that once a calculation involving several variables has been represented by a computation diagram or a nomograph, the user can often obtain solutions more quickly than is possible by computer. Two situations in higher education that warrant the use of diagrams and nomographs are as follows: whenever a routine calculation must be performed repeatedly, using different input values; and where a relatively simple analytical model is needed to calculate responses to "what if" questions posed in the form of values assigned to input variables. It is claimed that these devices equal the performance of a microcomputer, are more convenient to undertake, and can be used by all participants in a planning session. A mathematical model relating departmental workloads, student numbers, and faculty strength is developed. Diagrams include the following: a three-variable graph and a four-variable computational diagram for analyzing a technical/professional program. Seven reference texts on nomography and two on computation diagrams are identified. (SW)

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THE USE OF COMPUTATION DIAGRAMS AND NOMOGRAPHS IN HIGHER EDUCATION

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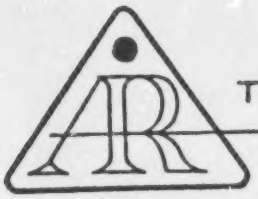

THE ASSOCIATION FOR INSTITUTIONAL RESEARCH

This paper was presented at the Twenty-Third Annual Forum of the Association for Institutional Research held at the Sheraton Centre in Toronto, Ontario, May 23-26, 1983. This paper was reviewed by the AIR Forum Publications Committee and was judged to be of high quality and of interest to others concerned with the research of higher education. It has therefore been selected to be included in the ERIC Collection of Forum papers.

D. R. Coleman, Chairman
Forum Publication
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Abstract

Within higher education circles, the effectiveness of graphic displays for presenting information to a general audience is well recognized. It is not, however, apparent that many researchers are aware of the value that graphs have when used as computational devices—a usage that is common within many engineering and physical science fields. The purpose of this paper is to demonstrate, with specific examples, how graphs can be used to quickly and easily make some calculations that are commonly used by institutional researchers. We shall point out how such graphs can be used within planning sessions much in the style of an interactive computer model to immediately answer the "what if" questions that arise. The technique of constructing such computational graphs (and nomographs) is outlined.

THE USE OF COMPUTATIONAL DIAGRAMS AND NOMOGRAMS IN HIGHER EDUCATION

Institutional researchers are well aware of the effectiveness of graphical displays and, where circumstances allow, often use them in place of tabular data displays. There are many such instances where the graphical display is an efficient and preferable means of communication. However, the same characteristics that make the graph a superior presentation device also ensures its usefulness and acceptability as a computational tool. Field engineers and scientists in many disciplines use graphs to make quick, repetitive and accurate calculations, a usage which has not been replaced by the hand calculator or microcomputer.

The reason that graphs are not used more for computation in higher education is probably due to the common practice of relating variables to geometrical dimensions. Once four or more variables must be linked, our limited ability to deal with a world beyond the three dimensional brings graphing to an end. This is may be illustrated by a simple example.

Suppose we consider an institution's tuition rate to be a decision variable (i.e., not fixed), and consider tuition revenue to be a function of <u>two</u> variables—enrollment and tuition rate. The standard graphical representation is three dimensional (Figure 1), which taxes both the person constructing the graph and the user.

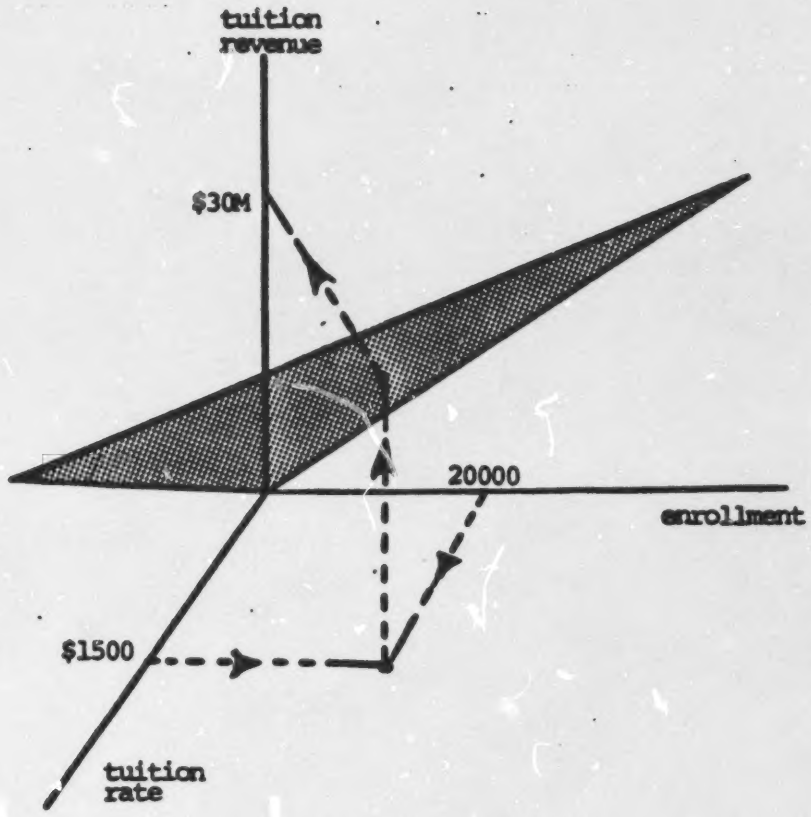


Figure 1 - A Three Variable Graph

As illustrated, the user enters the graph with an enrollment of 2,000 and a tuition rate of $1,500 and finds the point of intersection on the tuition-enrollment plane. Moving vertically from this point to the intersection within the tuition-enrollment-revenue surface and from here horizontally to the revenue axis we arrive at a revenue of $30M. The use of the three dimensional graph in this manner is more difficult than the more common two dimensional graph, but because of the linear relationships, an "answer" may be found via parallel lines. Once the relationship becomes non-linear, the surface develops a curvature and finding a reliable intersection point is no longer feasible.

Clearly this method, already cumbersome at three variables, becomes impossible once we consider tuition revenue to be a function of three variables, for now this type of approach requires a four dimensional graph.

Four Dimensional Computational Diagrams

All the preceding difficulties disappear once one abandons the traditional format of graphing variables along orthagonal axii. Figure 2 is such a graph, one which we choose to call a "computational diagram." This diagram relates four variables, namely workload in contact hours/FTE (on the left hand vertical axis), workload in student credit hours/FTE (on the "curved axis"), the number of students admitted each year (on the horizontal axis), and the number of faculty FTE (on the "flared diagonal" axis).

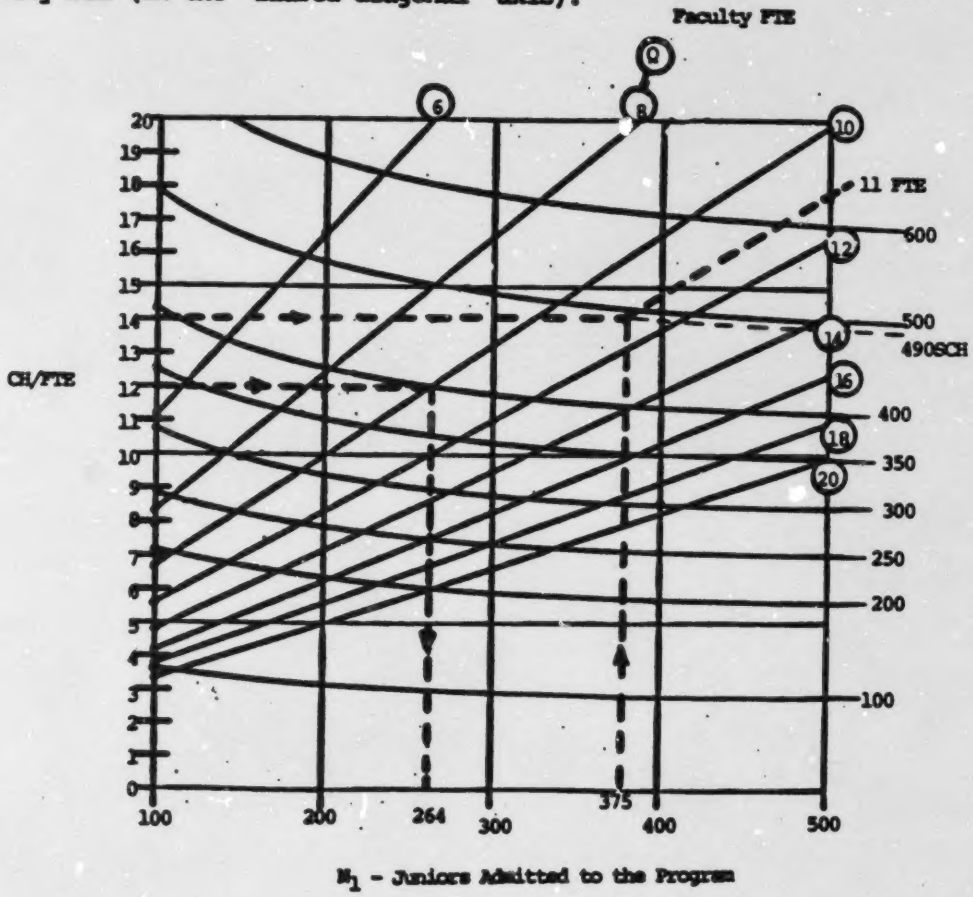


Figure 2. Four Variable Computational Diagram for Analyzing a Technical/ Professional Program

Figure 2 may be used in a number of ways. For instance, if the user wishes to maintain a 12 contact hour/FTE/term workload and has 10 faculty FTE, a horizontal line is drawn at the 12/CH/FTE/term level until it intersects with the 10 FTE "diagonal". From this intersection point a vertical line is dropped to the horizontal scale intersection indicating that about 264 juniors may be admitted each year. We also know that the workload level under these conditions will be approximately 400 SCH/FTE (because the intersection point lies near the 400 SCH/FTE scale curve.

This diagram may also be used to answer the question, "How many faculty do I need if I want to admit 375 new juniors/year and maintain a 14 contact hour/FTE workload?" We draw a horizontal line at 14 CH/FTE/term and a vertical line at 375 new juniors, the intersection point lies slightly more than half way between the 10 and 12 FTE diagonal scale lines indicating that about 11 FTE would be needed to maintain the required student population and workload. We also know that this situation would require a faculty workload of about 490 SCH/FTE.

Figure 2 was developed from a model based on a detailed analysis of a department's teaching model, laboratory limitations, course structure and staff configuration, the details of which are given in the next section.

A WORKLOAD MODEL

As undergraduate program preferences change it is often necessary to consider enrollment limitations for programs and departments within the university. This is especially true in times of economic troubles and budget reductions. In order to accomplish a realistic yet understandable analysis of departmental workloads, student numbers and faculty strength, a mathematical model relating these factors and others was developed.

The basic unit of this departmental model is the course. The workload contribution and student enrollment is modeled for each course offered by the department over a selected time interval, in this case the three term academic year. Several examples will illustrate this:

Example 1

EPD 200 is a four credit course offered each term. The lecture component of this course meets for three one hour sessions a week in sections of X_{200}; the laboratory component meets once a week for three hours in sections of X_{200L}. Half of the juniors and seniors in the program are usually enrolled in EPD 200 during any given year. CH and SCH are given by;

$$CH_{200} = \left(\frac{N_1 + N_2}{2}\right) \cdot \left(\frac{3}{X_{200}} + \frac{3}{X_{200L}}\right)$$

$$SCH_{200} = 4\left(\frac{N_1 + N_2}{2}\right)$$

where N_2 = number of seniors in the department's program and may be related to number of juniors by

$$N_2 = (1 - \alpha) N_1(t-1) \quad N_2 = (1 - \alpha) N_1(t-1)$$

where $N_1(t-1)$ is the number of juniors admitted the previous year and α is a drop-out fraction. (X_{200} may equal 60 and X_{200L} equal 20 students)

In Example 1 we have examined a course which is considered to be the "core" of the academic program and is only open to students admitted to the program.

Example 2

EPD 100 is a "service course" open to all university students. It is a three credit course taught in three one hour lectures each week. This course is taught in one large lecture section each term. The CH and SCH may be given by;

$$CH_{100} = 9$$

$$SCH_{100} = 541 + 9.47N_1$$

The SCH equation above was developed using historical data of service SCH vs. number of juniors in the program. A least-squares regression was used to fit a straight line to the (SCH, N_1) points thereby relating the service SCH to historical student demand for the program.

All courses in this particular department's program, core, service, elective and graduate, were modeled using the methods and combinations of methods outlined in the three examples above. Total contact hours and total SCH are then found simply by summing over all courses;

$$CH_{Tot} = CH_{100} + CH_{200} + \ldots.$$

$$SCH_{Tot} = SCH_{100} + SCH_{200} + \ldots.$$

Now that the two workload variables may be viewed as functions of admitted juniors, section sizes and other teaching model parameters, the workloads may be related to the FTE count of the faculty (Q). (All the course and staffing information are normalized for a three term academic year.) These are the relationships shown on Figure 2.

CONSTRUCTING A FOUR DIMENSIONAL COMPUTATIONAL DIAGRAM

The following is a step by step description of the procedure used to generate the diagram shown in Figure 2. This process may be generalized for any similar four dimensional relationship among variables.

1) Determine all model assumptions exactly. Set all model parameters (section sizes for example) at constant values except for the four parameters of interest: CH/FTE, SCH/FTE, N_1 and Q (N_1 = number of juniors admitted, Q = Faculty FTE).

2) Determine which two variables are to be used for the orthogonal axii. For our example, CH/FTE was placed on the vertical axis with a range of 0 to 20, and N_1 was placed on the horizontal axis with a range of 100 to 500 (N_1 min to N_1 max).

3) Determine the range of values to be shown on the Q scale; Qmin to Qmax (6 to 20 on Figure 2).

4) Set Q = Qmin, and incrementing N_1 between N_1min and N_1max calculate values for CH/FTE from the model's equations. This produces a series of (CH/FTE,N_1) points. Connect these points to produce the Qmin scale line.

5) Increment Q by a convenient value, Qinc (Qinc = 2 in Figure 2) and repeat Step 4. Continue this process through Q = Qmax. This produces the Q scale.

6) Determine the minimum and maximum values to be used for the SCH/FTE scale (SCH/FTEmin, SCH/FTEmax) and an appropriate increment between scale lines (SCH/FTEinc) (SCH/FTEinc = 50 and 100 in Figure 2).

7) Set SCH/FTE = SCH/FTEmin and Q = Qmin. Rearranging the model's equations, compute N_1 (SCH/FTEmin, Qmin); this is the point (N_1, Qmin), the intersection of the N_1 scale line with the Qmin scale line. Increment Q and recompute the point (N_1, Qmin + Qinc). Continue this process until N_1 = N_1max. Connect the (N_1,Q) points found in this step and this is the SCH/FTEmin scale line.

8) Increment SCH/FTE (SCH/FTEmin + SCH/FTEinc) and repeat the process in Step 7 and draw the second SCH/FTE scale line. Repeat this process until the SCH/FTE scale is complete.

Figures 3 and 4 are two more computational diagrams based on a different modeling procedure and different teaching models. These diagrams were successfully used in a planning negotiation between the provost and the involved dean and chairmen. The procedures, models and assumptions were agreed to by all parties involved in negotiating a solution. Figure 2 is a model of what could

be referred to as a technical/professional curriculum; Figure 3 was developed for a classical engineering program with almost no service component. Figure 4 is for a quantitative, engineering related discipline with no engineering laboratories but with a very large service component. These diagrams may be used exactly as was Figure 2. Clearly the underlying model may be quite complex in order to represent all the relevant variables; however, the resulting computational diagram is generally very simple in structure, and workload measures and enrollments may be quickly traded-off against one another to reach a solution that is acceptable to all participants in a planning session.

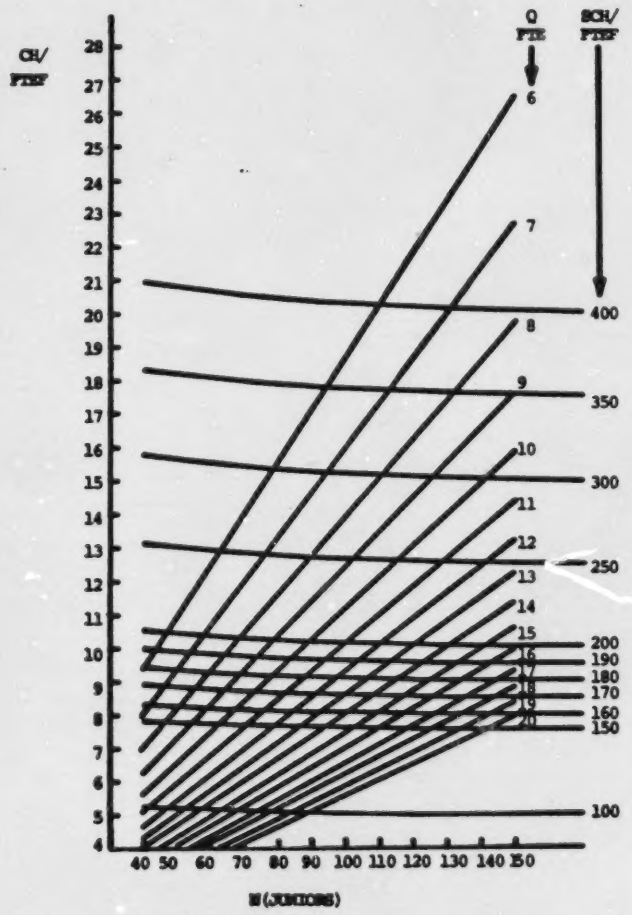


Figure 3 - Computation Diagram for an Engineering Program with No Service Component

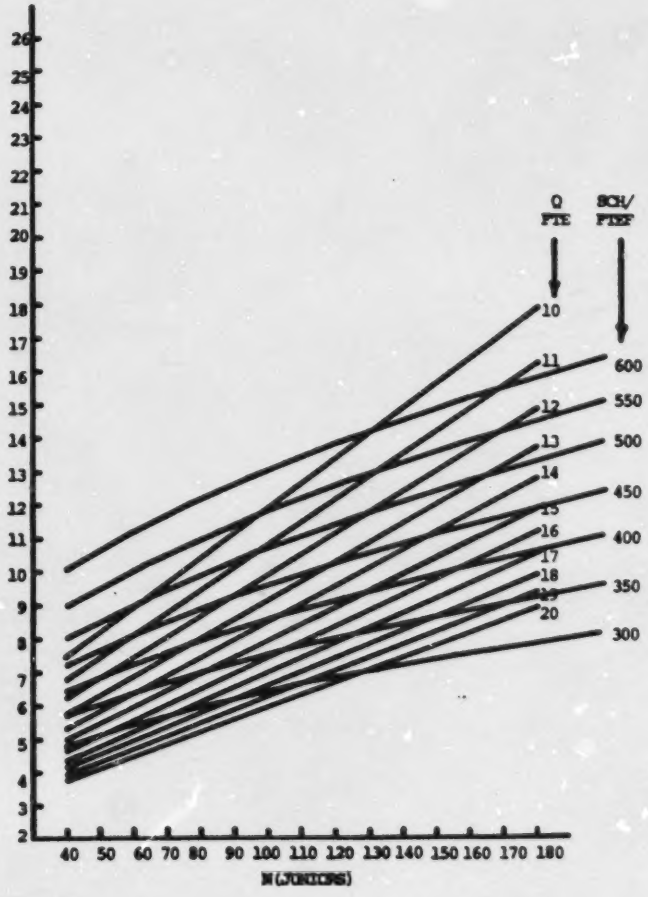


Figure 4 - Computation Diagram for an Engineering Discipline with No Laboratories

The value of such diagrams becomes apparent in those situations where one must make a series of repetitive calculations, when an equation must be solved for different variables or a complex computer simulation must be rerun. Such a situation may be exemplified by a planning session where the participants have only a vague idea of what should be done and prefer to arrive at an understanding by way of asking a series of "what if" questions. In the previous examples, planners could readily calculate their own answers to questions like, "If our 8 faculty were to teach 16 hours/week, how many juniors could we accept into the program?" or, "If we want to hold the teaching load to 12 hours/week how many faculty do we need to handle our 120 juniors?" The fact that a computational diagram can be easily used by <u>all</u> the participants to obtain nearly <u>immediate</u> answers to their questions is clearly preferable to the more common situation where all technical questions are referred to a specialist who responds days later. In this milieu, the computational graph serves the same purpose as a simple computer model that can be used interactively by all the participants provided with their own terminals.

Clearly diagrams of this nature are not limited by the complexity of the calculations as much as by the number of variables. Four or five variables can be accommodated within a readable format. Beyond that, the researcher must depend upon his ingenuity to make it understandable. However, most administrators and planners typically establish reference points by examining the effects of a few commonly used and accepted variables, a situation easily handled by our computational diagrams.

In the following sections, we enlarge upon the use of computational diagrams by considering a different type of graph—called a "nomograph"—which has characteristics that make it more useful than a computational diagram in certain instances.

NOMOGRAPHS

In the foregoing, we have shown that relatively simple calculations such as frequently occur in higher education administration can be easily performed using specially constructed graphs. We now illustrate yet another method to perform calculations, which involves a graph called a nomograph. At first glance, the nomograph may appear to be indistinguishable from the calculation graphs already discussed, but the underlying principles are quite different. In essence, a nomograph involves a set of numerical scales calibrated along straight lines which are, in most cases, parallel. Unknown values are found by using a straight edge to line up several given values and reading the unknown value located at the point of intersection of the straight edge and the scale of the unknown variable. Thus, in the nomograph of Figure 5, a given value of A=5 and B=10 would result in C taking on the value 400, where the dotted line represents a straightedge used to align points that are all related by a function linking A, B, and C.

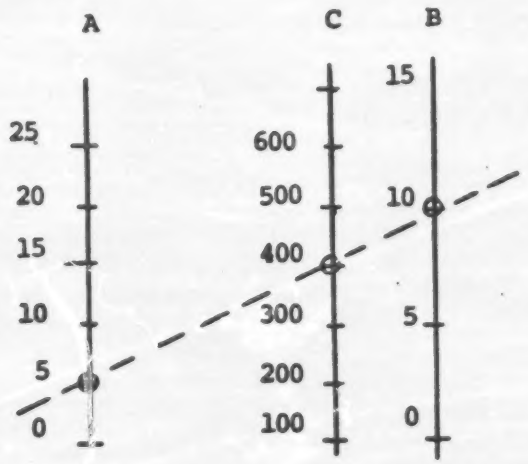


Figure 5 Reading a Nomograph

Nearly any calculation that can be represented by a calculation diagram can also be done with a nomograph or conversely. In general, nomographs work best when the number of variables are less than six and the mathematical relationships are relatively simple, i.e., involving arithmetic operations. Since the nomograph uses only the intersection of a line with other calibrated lines, the graph is free of the confusing clutter of curves and the horizontal and vertical grid lines associated with graph paper.

In addition, the nomograph technique enables one to easily obtain accurate results without the need to make rough interpolations between curves as is often the case with the use of calculation diagrams. A final advantage of the nomograph is that a step-by-step construction procedure can be formulated. In contrast, the greater versatility of the calculation diagram to handle more complex relationships involving many variables is gained only at the expense of a graph that is more difficult both to use as well as to construct.

As an illustration of the above mentioned advantages, consider the nomograph pictured in Figure 6b. This nomograph replaces the calculation diagram of Figure 6a, which represents the instructional model of a chemical engineering department. Using Figure 6a and given that the department has 110 entering juniors and a faculty of 9.5 FTE, the darker lines indicate that the ensuing workloads would be 250 SCH/FTE and 12.5 contact hrs/FTE. The same solutions can be gained from the nomograph in Figure 6b. The 9.5 FTE on the far left axis and the 110 juniors on the right axis are connected by a straight line that cuts the transversal axii at 12.5 and 250. A comparison of the two graphs will disclose some of the pros and cons discussed.

Note how quickly this procedure, using either graph, can be carried out with a few practice trials; in fact, one could perform these mechanical movements faster than key punching the input values of N and Q into a pre-

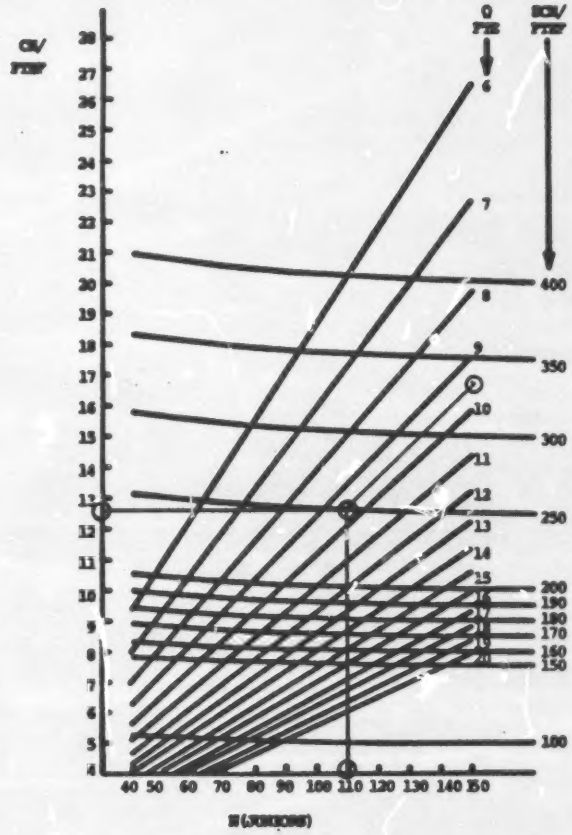


Figure 6a. Calculation Diagram

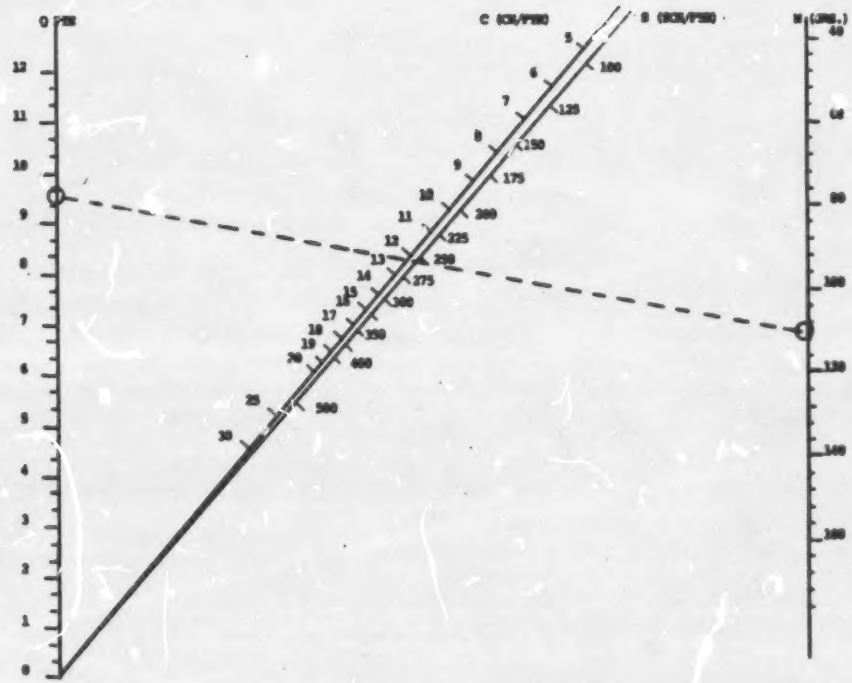


Figure 6b. Nomograph

programmed mini-computer or hand calculator. Certainly the nomograph is far faster than solving the original equations with a calculator. Finally, it should be clear that we do not have to always use N and Q to find SCH/FTE and CH/FTE. Given any two of the 4 variables (except SCH/FTE and CH/FTE), we can find the remaining two by constructing the tie line using the two given values.

CONSTRUCTING NOMOGRAPHS

Nomographs were invented in 1899 by a French mathematician, Maurice D'Ocagne, and quickly became a standard technique in the scientific and engineering fields. Most of the more useful nomographs were soon constructed and the practice of nomography dropped from usage as quickly as it had begun, disappearing from the engineering curriculum by 1930. As a result, the more useful textbooks on this technique are well over 50 years old.

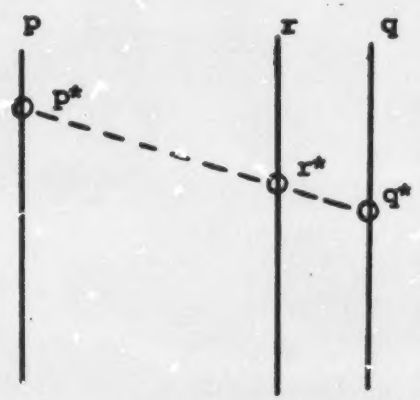
Most textbooks on nomography proceed by developing nomographs for certain fundamental equations. Once these standard forms are mastered, the technique is quickly generalized to more complex equations by viewing these equations as simply combinations of the fundamental forms. A quick summary of this approach is useful for gaining a general understanding of nomograph construction.

The first standard form is the equation $f_1(p) + f_2(q) = f_3(r)$ where f_1, f_2, and f_3 are functions of p, q, and r respectively. An example would be:

$$\underset{\uparrow}{(p-1)^2} + \underset{\uparrow}{q^2} = \underset{\uparrow}{(3r+2)}$$

$$f_1(p) \quad f_2(q) \quad f_3(r)$$

The characteristic nomograph associated with this equation is:

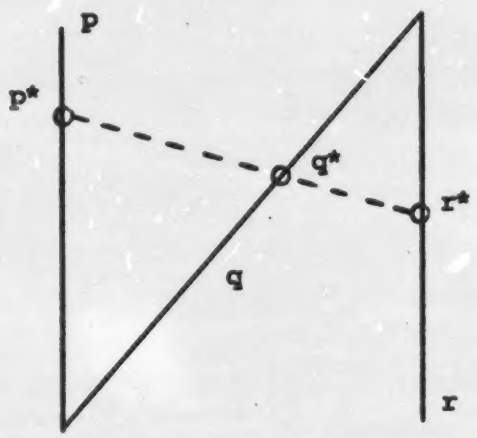


$$f_1(p^*)+f_2(q^*)= f_3(r^*)$$

Figure 7. Nomograph for the Basic Equation $f_1(p)+f_2(q) = f_3(r)$

The problem of construction becomes one of properly calibrating the p, r, q scales and determining the distance between them.

The second standard form is $f_1(p) \cdot f_2(q) = f_3(r)$ and has the characteristic form:



$$f_1(p^*) \cdot f_2(q^*)= f_3(r^*)$$

Figure 8. Nomograph for the Basic Form $f_1(p) \cdot f_2(q) = f_3(r)$

These two standard forms would suffice for a good many equations applicable to higher education. When used in combination, the possibilities are greatly extended, as will be demonstrated in the concluding example.

Example

Suppose we wish to construct a nomograph enabling us to find trade-offs between tuition increases and salary increases that will result in a balanced budget for a small state-supported college. Assume we are told that the current enrollment (E) is 10,000, the salary budget is $60 million, the tuition averages $2,000/year, additional revenues total $20 million and additional expenditures total to $10 million. The current State appropriation is $30 million.

Assume that enrollments, appropriations, tuition, and salary are all variables that can be changed.

Let: E = Most current enrollment

A = % change in the state appropriation

T = % change in the tuition

S = % change in the salary budget

If the model is to reflect a balanced budget we set:

revenue = expenditures

(tuition revenue + appropriation + other revenues) =

(salary budget + other expenditures)

$$E \times \$2{,}000(1+T) + \$30M(1+A) + \$20M = \$60M(1+S) + \$10M$$

Simplifying we get:

$$(2E) \cdot (1+T) + [10{,}000(3A-2)] = 60{,}000S$$

$f_1(E) \cdot f_2(T) \qquad f_3(A) \qquad f_4(S)$

Thus we want to construct a four variable nomograph for the equation of type:

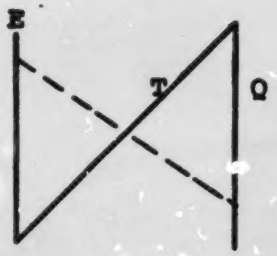
$$f_1(E) \cdot f_2(T) + f_3(A) = f_4(S) \qquad (1)$$

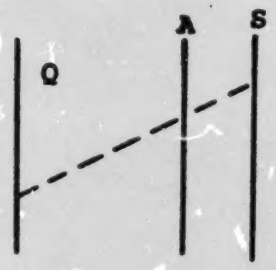
We handle this by writing it as two easy problems:

let: $Q = f_1(E) \cdot f_2(T)$ (2)

then equation 1 gives: $Q + f_3(A) = f_4(S)$ (3)

A nomograph can be constructed for each of these two equations:





$f_1(E) \cdot f_2(T) = Q$

$Q + f_3(A) = f_4(S)$

Figure 9 - Nomograph format for equations (2) and (3)

The final nomograph is formed by merging the above two nomographs by using a single Q axis.

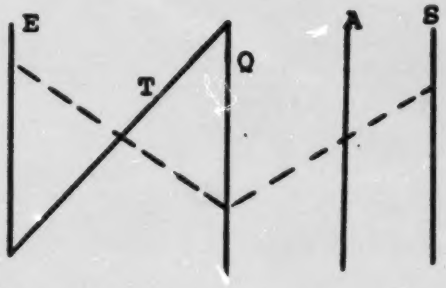


Figure 10 - Nomograph format for $f_1(E) \cdot f_2(T) + f_3(A) = f_4(5)$

Certainly the problem of plotting the precise position of the five scales and calibrating them remains. These details are not difficult to solve but cannot be explained in a paper of this length. The point we wish to make with the example is that useful nomographs can be constructed using minimal theory. Returning to the example, the completed nomograph appears as Figure 11.

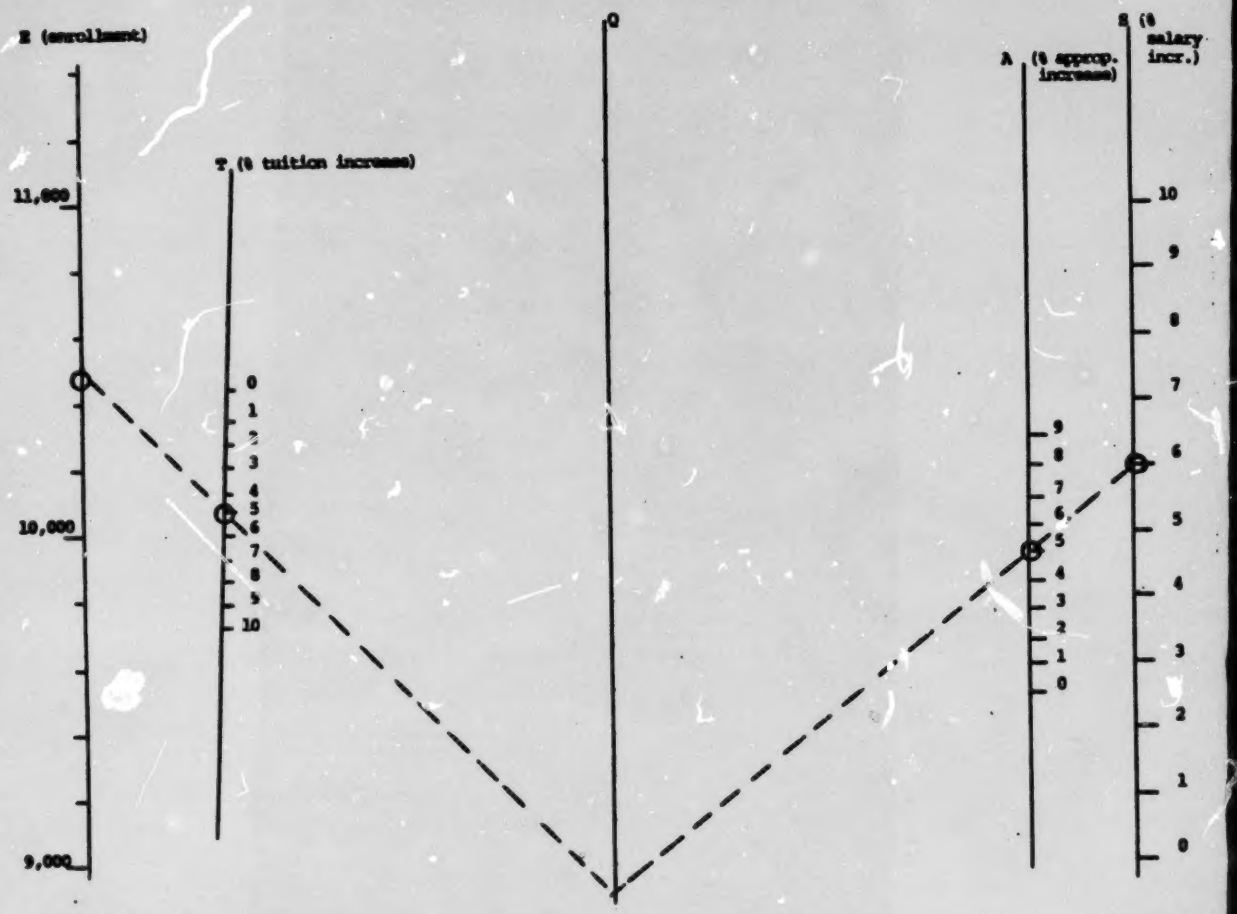


Figure 11. Nomograph of a State-Supported College Budget

We expected the T scale to be a transversal line from the E to the Q scale and yet it appears to be vertical. This is only because the E scale does not start at 0 but at 9,000; the slight incline of the T scale would cause it to cross the E scale at the 0 point, if extended.

A typical solution may proceed as follows. We are given that the expected appropriations will increase by 5% and that the enrollments will rise to 10,500. If we wish to increase salaries by 6%, what accompanying increase in tuition is required? The solution indicated by the dotted tie lines on Figure 6 is approximately 5%. (Numerical calculations produce a figure of 5.2%) Repeated trials can be performed to immediately answer similar "what if" questions and arrive at a consensus as to the proper course of action. Precise numerical calculations can then be made to refine the selected percentages.

SUMMARY

We have attempted to show that sophisticated calculators and microcomputers have not yet relegated all older computational techniques and devices to the dust bin. There are still certain situations in which older methods hold an advantage in both speed and convenience. The slowest step in any calculation using a calculator or computer is the human hand that keys in the data. As we have demonstrated, once a calculation involving several variables has been represented by a computation diagram or a nomograph, the user can often obtain solutions quicker than possible by computer. The first task is the identification of those special situations that warrant the use of diagrams and nomographs. In higher education administration there are at least two. One such instance occurs wherever a routine calculation must be performed repeatedly, using different input values. The other situation is more difficult to

recognize; it will most likely develop within a planning milieu, where a relatively simple analytical model is needed to calculate responses to "what if" questions posed in the form of values assigned to input variables. A fast response is essential. In this situation a computation diagram or nomograph reveals its full potential. Not only do these efficient devices equal the performance of a microcomputer, but they are more convenient to carry and, most importantly, they can be used by all participants in the planning session. More than one user has expressed delight over the simplicity and cleverness of these tools, an initial reaction that swiftly leads to a general acceptance of results. Experienced modelers will appreciate the psychological advantages that stem from a planner using a model in the form of a nomograph or computational graph to answer his/her own questions - a situation that is difficult to achieve if computer models or complex equations are used.

Once an alert administrator or researcher has discerned a situation conducive to the use of a nomograph or computation diagram, the task of constructing one remains. The construction is not difficult and can be readily mastered with the aid of one of the suggested texts.

SUGGESTED READING

Nomography

Adams, Douglas P. Nomography Conn.: Archon Book, 1964

Davis, Dale S. Nomography-Empirical Equations N.Y.: Reinhold, 1955

French, Thomas E. - Vierck, Charles J. Graphic Science N.Y.: McGraw-Hill, 1958

Lipka, Joseph Graphical & Mechanical Computation N.Y.: John Wiley & Sons, 1918

Marshall, William C. Graphical Methods N.Y.: McGraw-Hill, 1921

Swett, George W. Construction of Alignment Charts N.Y.: John Wiley & Sons, 1928

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Computation Diagrams

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